

Some Key Differences Between RM's Curriculum and Traditional Curricula Used in the United States

The General Approach

1. **Conceptual understanding and discovery learning.** New ideas are developed first by looking at examples and working special cases, with the student participating actively in the process. (The material is highly interactive.) After working through several examples, the student develops an intuitive sense of the general idea, rule, or method.
2. **Mathematical generalization.** Beginning with the Basic II curriculum, student learning is carried to the next level of abstraction: once the student has looked through concrete examples and developed a preliminary intuition for the general underlying idea, a rigorous statement of the idea is given and explained. Thus, the RM curriculum combines intuitive understanding of ideas on the one hand with their rigorous formulation on the other, and shows how the latter comes naturally from the former through the process of *mathematical generalization*.
3. **Rigor.** All of the rules, definitions, properties, and facts in the curriculum are presented as rigorously as possible. Math vocabulary is used consistently.
4. **Virtual manipulatives.** Students move around cubes (as well as ten-bars, hundred-plates, and thousand-cubes) when learning whole number place value, color in sections of shapes when studying fractions, and so on. This helps develop conceptual understanding without the logistical difficulties of physical manipulatives.
5. **Computational fluency.** The curriculum places a strong emphasis on computational fluency. This fluency is achieved not by rote memorization of algorithms, but rather as a natural extension of students' conceptual understanding of mathematical operations. The conceptual understanding and procedural fluency are developed concurrently, with each supporting the other.
6. **Depth.** The curriculum focuses strongly on the core areas of arithmetic, geometry, and algebra, focusing on depth over breadth.
7. **Spiraling and scaffolding.** The curriculum *spirals*, consistently using new topics to review previous topics; the sequential structure of mathematics – with each topic relying on what came before it – is brought to the foreground. On a related note, the curriculum pervasively uses *scaffolding*: each topic is presented with an eye to how it will be used in the future.
8. **Higher-order thinking skills.** Almost every topic has a bank of more difficult problems. Students who master the topic at the “basic” level receive problems at the “advanced” level; if successful with these, students work on problems at the “highest” level. The “advanced” and “highest” level problems are selected to develop higher-order reasoning skills: they are difficult not merely computationally, but also *conceptually*.
9. **Creative thinking.** Students solve math riddles, which are problems that require creativity and non-standard thinking. The goal of the RM curriculum is to afford students the opportunity to practice these skills.

Specifics

1. **Place value.** Thinking in terms of *place value* and the *composition of a number* is the key to understanding the meaning of whole number rounding, comparing whole numbers, and the four basic operations of arithmetic. Children understand the operations – and the associated standard algorithms – in terms of the structure of natural numbers. This knowledge forms a strong foundation for understanding common fractions and decimals.
2. **Mental math.** Mental math is used as a tool for strengthening students' conceptual understanding of the composition of numbers.
3. **Early use of algebraic concepts.** From the very earliest grades, the curriculum introduces algebraic concepts (such as algebraic expressions and solving equations). This prepares students for algebra down the road, and also supports the acquisition of certain arithmetical skills and concepts, such as the properties of multiplication and formulas for distance and price.
4. **Integration of geometry and arithmetic.** For example, perimeter is used as an additional model for the operation of addition – and an important one, since it works for fractions just as well as for natural numbers. Area is used to further develop the understanding of the operation of multiplication, as well as division. Thus, geometry becomes a natural, contributing part of the curriculum, rather than a disconnected topic.
5. **Use of geometry to visualize the operations.** In particular, the number ray is used to illustrate addition and subtraction, as well as the properties of these operations (such as the commutative property of addition and the property of subtracting a number from a sum). When the negative numbers are introduced in 6th grade, the number ray expands to the number line, creating a parallel between the arithmetical expansion of the number system and the geometric expansion of a ray into a line.
6. **Inverse operations.** Beginning with the Basic II curriculum, subtraction and division are introduced as inverse operations of addition and multiplication, respectively. This more mathematically mature definition significantly widens the class of problems that can be approached with these operations. For example, finding an unknown length of a rectangle given the area and the width now becomes a natural *model* for division. Such models are particularly important in the Basic II and Basic III curricula, since the conventional models for the operations (e.g., “5 groups of 4 objects” for 5×4) no longer make sense for fractions.
7. **Numerical expressions.** Numerical expressions are extensively used from the very beginning of the Basic I curriculum, and act as a stepping stone to work with algebraic expressions, which in turn lead to equations.
8. **Properties of the operations.** Students learn to use the properties of the operations (e.g., the distributive property of multiplication over addition) to transform numerical and algebraic expressions. This allows them to understand and actively apply these important properties (which in traditional U.S. curricula are taught by rote memorization or entirely omitted), and prepares students for algebra, where without

solid skills in algebraic transformations, they will not be able to set up and solve equations.

9. **“Convenient calculation.”** Students use the properties of the operations for convenient calculation. This means that students are given a numerical expression to evaluate which would be very difficult to evaluate using the order of operations, but quite simple to evaluate if the student first applies certain properties of the operations. This not only serves as a way to make students use the properties, but also develops number sense and mental math skills.
10. **Decimals are fractions.** Decimals are taught as a special case (and notation) of common fractions, rather than a separate entity altogether.
11. **Word problems as models.** Word problems are used extensively, but not just for their own sake. Rather, word problems are a powerful tool for helping students understand the meanings of the mathematical operations. By seeing an operation – say, subtraction – illuminated from different angles and in different contexts, students evolve an abstracted understanding of this operation. The problem sets given in the RM curriculum are carefully selected to achieve this effect.

The History

The Reasoning Mind curriculum is essentially based on the Russian curriculum developed in the mid-20th century. This curriculum then went into the foundations of the Chinese curriculum, which in turn was taken as the foundation of the curriculum used in Singapore. Remarkably, most countries that perform well on international math assessments (including Hungary, Bulgaria, Romania, Poland, Belgium, and many others) have a national curriculum that has grown out of the Russian program. Thus, the program has proven itself successful with billions of students.

There is consensus among most Russian educators about the reasons for the success of this approach. Mathematics in this approach is taught in a connected, spiraling fashion; this means that every topic is covered in the context of preceding topics, and especially if they are conceptual prerequisites to the topic. Problem solving is the key learning activity: theoretical material is given importance, but true conceptual understanding and intuitive command of mathematical ideas comes from solving a series of carefully selected problems. Another reason for the method's success is that over time, a huge amount of knowledge has been amassed by practitioners of the method as to how certain topics should be best presented to students. Finally, the method places logical, creative, and non-standard thinking as its goal. Students study mathematics to learn how to think, and educators constantly keep this goal in mind.

The Russian program has never been widely practiced in the U.S. Unfortunately, traditional methods of teaching math in this country haven't been effective. Here is what Dr. William H. Schmidt, the Research Coordinator of the U.S. Third International Mathematics and Science Study, writes about the differences between American curricula and curricula in high-achieving countries:

“Third International Mathematics and Science Study results suggest that the top achieving countries have coherent, focused and demanding mathematics curricula. What would a coherent curriculum look like? A coherent curriculum leads students through a sequence of topics and performances over the grades that reflects the logical and sequential nature of knowledge in mathematics. Such a curriculum helps students to

move from particular knowledge and skills toward an understanding of deeper structures, more complex ideas and mathematical reasoning including problem solving. For example, students should be expected to master the basic concept of number and basic computational skills in the early grades before they tackle more difficult mathematics.

What does the US curriculum look like? The US curriculum as reflected in many of the states' standards and in our nation's textbooks tends to reflect an arbitrariness where topics appear somewhat haphazardly throughout the grades. For example, teachers are expected to introduce relatively advanced mathematics in the earliest grades before students have had an opportunity to master basic concepts and computational skills. Secondly, the curriculum continues to focus on basic computational skills through grade eight and perhaps beyond. I would argue that if the logic of mathematics is not transparent to students, then it becomes difficult for them to develop a deep understanding that would lead to higher achievement.

What does a focused and rigorous curriculum look like in the top achieving countries? The number of topics that children are expected to learn at a given grade level is relatively small, permitting a thorough and deep coverage of each topic. For example, nine topics are the average number intended in the second grade. The US by contrast expects second grade teachers to cover twice as many mathematics topics. The result is a characterization of the US curriculum as a mile wide and an inch deep.

Coherent standards move from the simple to the complex. By the middle grades the top achieving countries do not intend that children should continue to study basic computation skills but rather that they begin the transition to the study of algebra, including linear equations and functions, geometry and even in some cases, basic trigonometry. By the end of eighth grade in these countries children have mostly completed US high school courses in algebra I and geometry. By contrast, most US students are destined to mostly continue the study of arithmetic. In fact, we estimate that at the end of eighth grade US students are some two or more years behind their counterparts around the world."

The Russian curriculum develops both concepts and skills, and emphasizes both discovery learning and mathematical rigor. For this reason, it is widely respected by U.S. math curriculum specialists from across the spectrum of philosophies. RM's supporters include prominent leaders of both the "constructivist" and the "back to basics" schools of thought – as well as educators in between.

Some interesting comments concerning the Russian curriculum were made by Dr. R. James Milgram, Professor Emeritus at Stanford University. Dr. Milgram is recognized as one of the nation's leading authorities on math education, and his comments can be watched in a movie – called *The Reasoning Mind Story, Episode II* – which is [available online on Reasoning Mind's website](#).

Reasoning Mind took this Russian curriculum as the basis of its approach and modified it to fit the cultural needs of American students. Moreover, RM used the flexibility of the Internet to enrich the curriculum with animations and an extremely high level of interactivity. The resulting program is very different in the mode of presentation from the Russian curriculum, but at its core one still finds the solid, time-tested instructional practices of the Russian program.