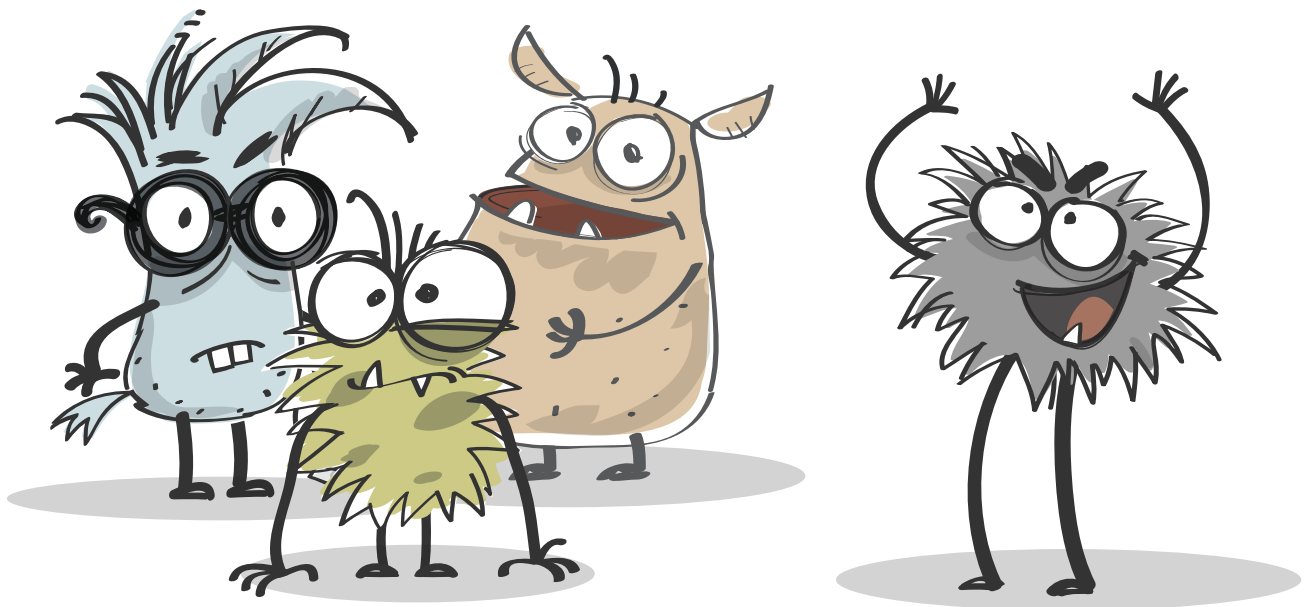


Foundations for Middle School 6th Grade Syllabus



UNIT 1: Review of Mixed Numbers, Decimals, and Fractions (Lessons 1-19)

SUMMARY

Unit 1 introduces students to the Reasoning Mind system while reviewing essential arithmetic skills. These skills include reading, writing, comparing, adding, subtracting, multiplying, and dividing various types of numbers: whole numbers, decimals, and fractions and mixed numbers with common denominators. Students review the fact that a single number may be written in more than one way. Students also review related terminology: sum, summand, minuend, subtrahend, difference, numerator, denominator, whole part, fractional part, place value, etc. This terminology will allow students to have discussions using precise mathematical language and understand precisely formulated rules. Other topics include: analyzing the givens in a word problem, solving word problems requiring one operation, solving equations with one operation, percentages, and mental math.

ASSESSMENTS

Quiz Lessons: 7, 10, 13, 17

Exam Lesson: 19

ESSENTIAL LEARNING OBJECTIVES¹

- ✓ Add, subtract, multiply and divide natural numbers and decimals using column operations
- ✓ Add and subtract fractions with the same denominator
- ✓ Solve equations with one variable and one operation
- ✓ Solve simple word problems with percentages

EXTENSION LEARNING OBJECTIVES²

- ✓ Know that addition or multiplication of several terms can be done in any order and use this property to make calculations easier
- ✓ Analyze the givens in a word problem and relationships between them, such as "X is Y times more/less than Z" and "in total," and use that to solve word problems requiring a single operation with decimals

¹Essential Learning Objectives for each unit indicate knowledge, skills and abilities which students should master in order to progress through the curriculum successfully. Mastery of these skills is measured on quizzes and the first three questions of each unit exam.

²Extension Learning Objectives for each unit indicate knowledge, skills and abilities that challenge students to apply and extend previously learned material. Students have the opportunity to demonstrate mastery of these skills on the last two questions of each unit exam.

UNIT 2: Divisibility (Lessons 20–38)

SUMMARY

Unit 2 introduces the concepts of factors and multiples, and rules are given for identifying numbers that are divisible by 2, 3, 5, 9, and 10. Students are then introduced to the concepts of prime numbers, composite numbers, greatest common factors (GCF), and least common multiples (LCMs). Two different methods are given for finding GCF and LCMs: a “brute force” method which is straightforward but somewhat work intensive, and a method based on prime factorization, which is conceptually more advanced but easier to carry out. GCF and LCMs play an essential role in Units 3-7, where they provide the key to working with fractions that have different denominators.

ASSESSMENTS

Quiz Lessons: 23, 27, 35

Exam Lesson: 38

ESSENTIAL LEARNING OBJECTIVES

- ✓ Use divisibility tests on numbers up to 1000
- ✓ Find factors and multiples of a two-digit number
- ✓ Find the GCF and the LCM of a pair of 1- or 2-digit numbers, in particular, when the numbers are given as products of prime numbers

EXTENSION LEARNING OBJECTIVES

- ✓ Find the value of an expression with two operations and parentheses involving decimals.
- ✓ Use divisibility tests on numbers up to 9999 to find a number satisfying a certain additional condition (for instance, having a certain digit)

UNIT 3: Comparing, Adding, and Subtracting Fractions with Different Denominators (Lessons 39–54)

SUMMARY

Unit 3 begins by using number lines to illustrate the concept of equivalent fractions. Then, the Fundamental Property of Fractions is introduced, which formalizes the algebraic relationship between equivalent fractions. Students apply this relationship to simplify fractions (using GCFs) and to bring two fractions to a common denominator (using LCMs). Once these skills are firmly in place, students can use simple rules to compare, add, and subtract fractions with different denominators. Students also begin solving equations of increasing complexity; in this unit, students solve equations involving two operations instead of just one.

ASSESSMENTS

Quiz Lessons: 41, 44, 47, 50, 52
Exam Lesson: 54

ESSENTIAL LEARNING OBJECTIVES

- ✓ Compare, add, and subtract fractions with different denominators
- ✓ Write fractions in lowest terms (“reduce fractions”)
- ✓ Know to not give reducible or improper fractions as the final answer

EXTENSION LEARNING OBJECTIVES

- ✓ Add or subtract fractions with different denominators to solve a word problem
- ✓ Add or subtract fractions with different denominators to solve an equation in which one of the terms is a natural number or a decimal

UNIT 4: Comparing, Adding, and Subtracting Mixed Numbers (Lessons 55–63)

SUMMARY

In Unit 4, students build on work from Unit 3 by comparing, adding, and subtracting mixed numbers. Students learn how to decompose a number's whole part when a subtraction problem requires "borrowing." (For example, to calculate $5 - \frac{1}{3}$, students decompose 5 into $4 + \frac{3}{3}$, subtract $\frac{1}{3}$ from $\frac{3}{3}$, and then add the resulting $\frac{2}{3}$ to 4.) Students are also introduced to an important algebraic concept: evaluating an expression with one variable for given values of that variable.

ASSESSMENTS

Quiz Lessons: 58, 60

Exam Lesson: 63

ESSENTIAL LEARNING OBJECTIVES

- ✓ Compare, add, and subtract mixed numbers with different denominators
- ✓ Write the fractional part of a mixed number as a proper irreducible fraction
- ✓ Solve equations with a single addition or subtraction involving fractions or mixed numbers

EXTENSION LEARNING OBJECTIVES

- ✓ Add or subtract mixed numbers with different denominators to solve a word problem
- ✓ Add and subtract fractions and mixed numbers to evaluate a letter expression that might also contain a decimal

UNIT 5: Multiplying Fractions (Lessons 64–77)

SUMMARY

In Unit 5, students learn to multiply fractions and mixed numbers. These new skills are motivated using real-world situations and geometric examples. Early exercises involve fractions that can be easily represented as decimals, allowing students to check their answers by multiplying the numbers in decimal form. The associative and distributive properties of multiplication are reviewed, and the class verifies that these properties remain true for fractions and mixed numbers. Students are introduced to the concept of like terms and begin solving equations that require combining like terms.

ASSESSMENTS

Quiz Lessons: 67, 71

Exam Lesson: 77

ESSENTIAL LEARNING OBJECTIVES

- ✓ Perform a single multiplication of natural numbers, decimals, fractions and mixed numbers in any combinations
- ✓ Find a fraction of a given number or quantity as a standalone task or to solve a word problem requiring a single operation
- ✓ Find a percentage of a given number or quantity via converting the percents to a decimal as a standalone task or to solve a word problem requiring a single operation

EXTENSION LEARNING OBJECTIVES

- ✓ Simplify expressions with like terms having fractional coefficients and apply this skill to solve equations and evaluate expressions

UNIT 6: Dividing Fractions (Lessons 78–85)

SUMMARY

Unit 6 begins by introducing the concept of a reciprocal, and students learn to find the reciprocals of whole numbers, mixed numbers, and fractions. This concept is then combined with the content of Unit 5 to formulate the rule for dividing fractions: to divide a number by a fraction, multiply that number by the reciprocal of the fraction. Students then solve equations and word problems that require dividing by a fraction.

ASSESSMENTS

Quiz Lessons: 81, 83

Exam Lesson: 85

ESSENTIAL LEARNING OBJECTIVES

- ✓ Perform a single division of natural numbers, decimals, fractions and mixed numbers in any combinations
- ✓ Divide fractions and mixed numbers to solve equations

EXTENSION LEARNING OBJECTIVES

- ✓ Evaluate expressions with up to 3 arithmetic operations, with or without parentheses, with all forms of numbers known to students to date (natural numbers, decimals, fractions, and mixed numbers)
- ✓ Find fractions of given quantities to solve word problems requiring multiple operations
- ✓ Find percentages of given quantities to solve word problems requiring multiple operations

UNIT 7: Problems with Fractions and Fractional Expressions (Lessons 86–95)

SUMMARY

Having completed Units 3-6, students can now add, subtract, multiply, and divide any combination of whole numbers, decimals, fractions, and mixed numbers. Unit 7 solidifies these computational skills, as students solve a wide variety of equations and word problems. Special focus is given to two types of problems: 1) finding a fraction or percentage of a known number, and 2) finding a number, if a fraction or a percentage of that number is known. Students are also introduced to fractional expressions, and learn how to evaluate them.

ASSESSMENTS

Quiz Lessons: 89, 92

Exam Lesson: 95

Note: Exam 7 covers units 5, 6, and 7, and has problems on learning objectives from each of these units.

ESSENTIAL LEARNING OBJECTIVES

- ✓ Evaluate fractional expressions in different cases (when the fractional expression is written as a fraction or as a quotient, with its components being natural numbers, decimals, fractions, or mixed numbers)
- ✓ Find a number or quantity from a fraction or a percentage of it
- ✓ Find a number or quantity from a fraction of it to solve a word problem

EXTENSION LEARNING OBJECTIVES

- ✓ Evaluate expressions with up to 3 arithmetic operations, with or without parentheses, with all forms of numbers known to students to date (natural numbers, decimals, fractions, and mixed numbers)
- ✓ Find a number or a quantity from a fraction of it to solve a word problem (a fraction may be described in words, i.e. “a half” or “a third”), including cases when the fraction given is of another fraction of the quantity in question
- ✓ Simplify expressions with like terms to solve equations

UNIT 8: Ratios and Proportions(Lessons 96–108)

SUMMARY

Unit 8 introduces students to ratios and proportions. These concepts are motivated by real-world examples, then formalized in the language of fractions and quotients. Students connect these concepts to material from Unit 7 by analyzing the relationship between fractions and percentages. Students learn how to find an unknown quantity in a proportion, and use this skill to solve equations and word problems. Two types of proportional relationships are introduced: direct proportionality and inverse proportionality. Students practice identifying these relationships in word problems and creating corresponding proportions to solve them.

ASSESSMENTS

Quiz Lessons: none

Exam Lesson: 108

ESSENTIAL LEARNING OBJECTIVES

- ✓ Find the unknown term in proportions
- ✓ Find percentages of given quantities via proportions to solve word problems
- ✓ Solve word problems on direct proportionality

EXTENSION LEARNING OBJECTIVES

- ✓ Solve word problems on inverse proportionality
- ✓ Evaluate expressions with up to 4 operations, parentheses and all forms of numbers known to students to date

UNIT 9: Additional Topics (Lessons 109–113)

SUMMARY

Unit 9 applies skills from the previous Units to several additional topics: scale and conversion between units, the geometry of circles and the number pi, and the golden ratio. These topics give students a small taste of the variety of mathematics.

ASSESSMENTS

Quiz Lessons: 113

Exam Lesson: none

ESSENTIAL LEARNING OBJECTIVES

- ✓ Solve problems involving scale
- ✓ Find the circumference of a circle
- ✓ Find the area of a disk

UNIT 10: Cumulative Review (Lessons 114–122)

SUMMARY

Unit 10 reviews the material from Units 1-8 to prepare students for the final exam. (Material from Unit 9 does not appear on the final exam.)

ASSESSMENTS

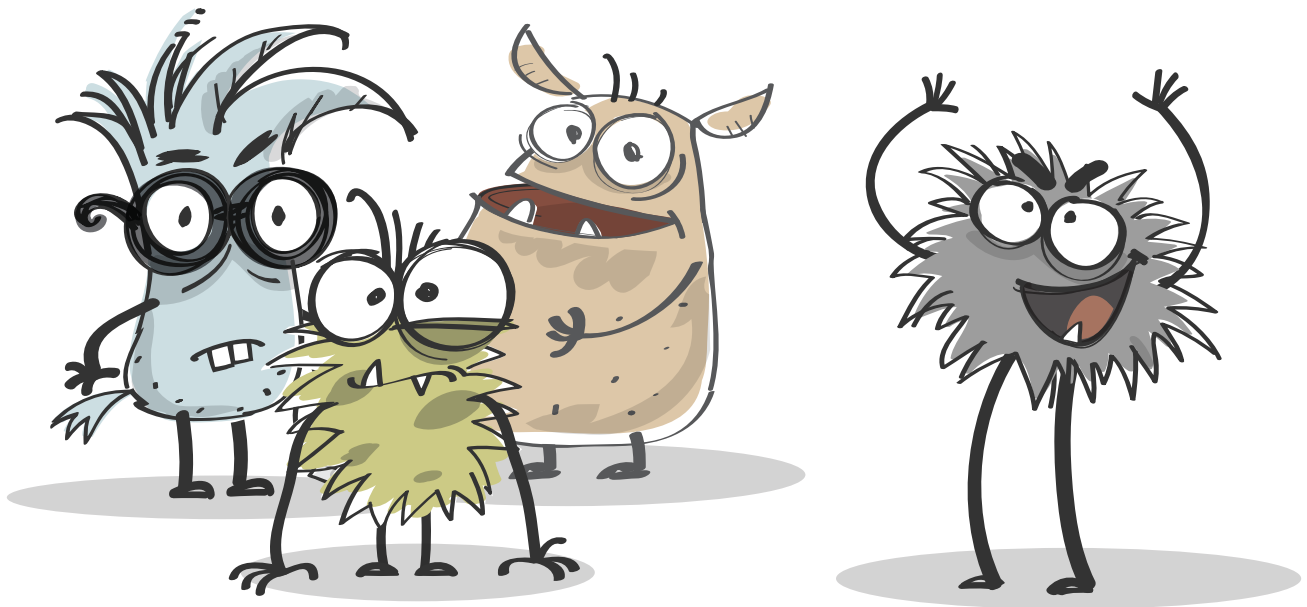
Quiz Lessons: 114, 115, 116, 118, 120

Final Exam Lesson: 122

ESSENTIAL LEARNING OBJECTIVES

- ✓ Evaluate expressions with up to 3 operations, with parentheses, that include natural numbers and decimals
- ✓ Add, subtract, multiply and divide fractions to solve equations
- ✓ Find a number or quantity from a percentage of it to solve a word problem
- ✓ Find factors and multiples of a given natural number
- ✓ Solve problems requiring 2 operations with any forms of numbers known to students to date

Foundations for Middle School 7th Grade Syllabus



UNIT 1: Review of Mixed Numbers, Decimals, and Fractions (Lessons 1-10)

SUMMARY

Unit 1 reviews essential arithmetic skills: comparing, adding, subtracting, multiplying, and dividing natural numbers, decimals, fractions, and mixed numbers. We also review related terminology: summand, sum, factor, product, minuend, subtrahend, difference, dividend, divisor, quotient. This terminology will allow students to have discussions using precise mathematical language and understand precisely formulated rules. Other topics include: proportions, percentages, representing natural numbers on the number line, solving simple equations, and solving word problems using shorthand.

ASSESSMENTS

Quiz Lessons: 4

Exam Lesson: 10

ESSENTIAL LEARNING OBJECTIVES¹

- ✓ Evaluate a numerical expression with 3 operations involving decimals
- ✓ Solve simple equations by performing arithmetical calculations with common fractions and mixed numbers
- ✓ Use a proportion to find a number if some percentage of that number is known

EXTENSION LEARNING OBJECTIVES²

- ✓ Simplify a numerical expression using the distributive property, and evaluate the expression by performing calculations with decimals and fractions
- ✓ Solve problems about parts of a whole involving 2 operations

¹Essential Learning Objectives for each unit indicate knowledge, skills and abilities which students should master in order to progress through the curriculum successfully. Mastery of these skills is measured on quizzes and the first three questions of each unit exam.

²Extension Learning Objectives for each unit indicate knowledge, skills and abilities that challenge students to apply and extend previously learned material. Students have the opportunity to demonstrate mastery of these skills on the last two questions of each unit exam.

UNIT 2: Positive and Negative Numbers (Lessons 11-24)

SUMMARY

Unit 2 uses number lines to introduce negative numbers, opposite numbers, and absolute value. Positive and negative numbers are compared geometrically, using number lines, and algebraically, using absolute values. Finally, positive and negative numbers are used to represent movement along the number line, which provides students with a common-sense framework for understanding the rules of addition and subtraction involving negative numbers, which will be introduced in Unit 3.

ASSESSMENTS

Quiz Lessons: 12, 15, 17, 21

Exam Lesson: 24

ESSENTIAL LEARNING OBJECTIVES

- ✓ Find the coordinate of a point depicted on a number line
- ✓ Plot a point on a number with given coordinates, which may be
 - Ⓐ Integers (between -20 and 20)
 - Ⓑ Decimals (with fractional part $.1$, $.2$, $.25$, $.3$, $.4$, $.5$, or $.75$)
 - Ⓒ Fractions (with denominator 2 , 3 , 4 , or 6)
- ✓ Identify the integers in a list of numbers
- ✓ Find the opposite of a given number, and evaluate expressions like $-(-5)$
- ✓ Find numbers whose absolute value has been given, and evaluate expressions containing absolute values
- ✓ Make the following comparisons, and write the result as an inequality:
 - Ⓐ Compare any positive number to any negative number
 - Ⓑ Compare any number to 0
 - Ⓒ Compare a positive integer to any other positive number, and compare two positive decimals, two positive fractions, or two positive mixed numbers
 - Ⓓ Compare a negative integer to any other negative numbers, and compare two negative decimals, two negative fractions, or two negative mixed numbers

There are no Extension Learning Objectives for this unit.

UNIT 3: Adding and Subtracting Positive and Negative Numbers (Lessons 25-37)

SUMMARY

Unit 3 uses the concept of absolute value to introduce rules for adding and subtracting positive and negative numbers. These rules are applied to mixed numbers, decimals, and fractions, and extensive computational practice integrates the new rules with the skills and concepts from Units 1 and 2. Work/productivity/time word problems are introduced.

ASSESSMENTS

Quiz Lessons: 28, 30, 32, 34

Exam Lesson: 37

ESSENTIAL LEARNING OBJECTIVES

- ✓ Mentally add and subtract positive and negative numbers, including integers, decimals, fractions, and mixed numbers
- ✓ Apply the previous skill to solve simple equations and word problems

EXTENSION LEARNING OBJECTIVES

- ✓ Evaluate a letter expression, working with positive and negative numbers
- ✓ Apply an understanding of the meaning of positive negative numbers when solving a word problem

UNIT 4: Multiplying and Dividing Positive and Negative Numbers (Lessons 38-50)

SUMMARY

Unit 4 begins with multiplication and division of positive and negative numbers. Then, the concept of rational numbers is introduced, which unifies all previous types of numbers (natural numbers, integers, mixed numbers, fractions, and decimals.) We introduce the formal properties of operations with rational numbers, including the commutative, associative, and distributive properties, and the various properties of 0, 1, opposites, and inverses.

ASSESSMENTS

Quiz Lessons: 44, 48

Exam Lesson: 50

ESSENTIAL LEARNING OBJECTIVES

- ✓ Mentally multiply and divide positive and negative numbers and decimals
- ✓ Apply the previous skill when solving equations and evaluating numerical expressions

EXTENSION LEARNING OBJECTIVES

- ✓ Use the properties of operations to evaluate a numerical expression in a convenient way
- ✓ Apply the aforementioned operations while solving word problems

UNIT 5: Parentheses, Coefficients, and Like Terms (Lessons 51-61)

SUMMARY

To prepare students for more complex equations and expressions, Unit 5 introduces rules for removing parentheses and combining like terms. Students are also introduced to word problems where quantities are given as variables, and asked to create expressions to describe related quantities.

ASSESSMENTS

Quiz Lessons: 53, 55

Exam Lesson: 61

ESSENTIAL LEARNING OBJECTIVES

- ✓ Use the rules for removing parentheses to simplify expressions and evaluate numerical expressions
- ✓ Recognize and combine like terms

EXTENSION LEARNING OBJECTIVES

- ✓ Use the associative property of multiplication to simplify letter expressions, and use the distributive property and the rule for removing parentheses to simplify letter expressions
- ✓ Make an expression from the text of a word problem

UNIT 6: Solving Equations (Lessons 62-68)

SUMMARY

Unit 6 begins with the introduction of two rules for transforming equations without changing their solutions. Students then apply all previously learned skills to solve equations requiring significant algebraic manipulation. Students are introduced to the idea of solving word problems by assigning a variable to an unknown quantity, setting up and solving an equation, and using that solution to answer the word problem.

ASSESSMENTS

Quiz Lessons: 65

Exam Lesson: 68

ESSENTIAL LEARNING OBJECTIVES

- ✓ Solve equations by moving summands
- ✓ Find the root of an equation
- ✓ Remove parentheses while working with natural numbers, fractions, and decimals
- ✓ Write mathematical sentences as equalities

EXTENSION LEARNING OBJECTIVES

- ✓ Write equations based on the text of a word problem

UNIT 7: The Coordinate Plane (Lessons 69-79)

SUMMARY

Unit 7 takes a break from algebra and shifts the focus to geometry: parallel and perpendicular lines are introduced and used in the construction of the coordinate plane. Students get extensive practice writing the coordinates of points and plotting points given their coordinates. Graphs are then introduced, and the new skills from this unit are used to answer word problems involving graphs.

ASSESSMENTS

Quiz Lessons: 73, 77

Exam Lesson: 79

ESSENTIAL LEARNING OBJECTIVES

- ✓ Plot points when the coordinates are nonzero integers
- ✓ Determine the coordinates of a point when the coordinates are integers
- ✓ Understand how the quantities in a word problem are related to the quantities in a coordinate plane (for example, interpret the y -coordinate 5 as the temperature 5°C)
- ✓ Determine the values of marked intervals on the axes of a graph

EXTENSION LEARNING OBJECTIVES

- ✓ Evaluate an expression that requires more than 3 steps and involves fractions or decimals
- ✓ Find the coordinates of a point when those coordinates depend on other numbers
- ✓ Plot points when one of the coordinates is 0

UNIT 8: Expressions (Lessons 80-95)

SUMMARY

Unit 8 begins by touching on bar graphs, then returns the focus to algebra. Students practice reading and evaluating numerical expressions, and composing numerical expressions that represent quantities in word problems. Then we transition to expressions with variables, and evaluating and comparing these expressions for given values of the variables. Students continue to gain familiarity with the idea of using letter expressions to solve word problems. The unit ends with the concepts of identities, identity transformations, and identically equal expressions. These concepts shed light on the various algebraic transformations introduced this year: using properties of operations, removing parentheses, and combining like terms.

ASSESSMENTS

Quiz Lessons: 83, 86, 91

Exam Lesson: 95

ESSENTIAL LEARNING OBJECTIVES

- ✓ Evaluate numerical expressions with three operations
- ✓ Evaluate expressions with two variables for given values of the variables
- ✓ Simplify expressions containing variables by
 - Ⓐ Using the associative and distributive properties of multiplication
 - Ⓑ Combining one or two groups of like terms
 - Ⓒ Removing parentheses

EXTENSION LEARNING OBJECTIVES

- ✓ Compare the values of expressions for given values of the variables
- ✓ Use expressions with variables to solve word problems
- ✓ Make a numerical expression from the text of a word problem

UNIT 9: Linear Equations (Lessons 96-104)

SUMMARY

Unit 9 expands on the equation-solving skills that students have developed in the previous units. The concepts of “equation,” “root,” “solving an equation,” and “linear equation” are formally defined, and the different types of linear equations are thoroughly analyzed. Then, the focus shifts to solving word problems by assigning a variable to an unknown quantity, using the problem text to set up and solve an equation, and answering the word problem using that solution.

ASSESSMENTS

Quiz Lessons: none

Exam Lesson: 104

ESSENTIAL LEARNING OBJECTIVES

- ✓ Check if a given number is a root of a given equation
- ✓ Combine like terms in equations
- ✓ Turn a given equation into a linear equation by
 - Ⓐ Moving summands
 - Ⓑ Removing parentheses
 - Ⓒ Using the distributive property
- ✓ Find the root of a linear equation (in the form $ax = b$) by dividing by the coefficient ($x = \frac{b}{a}$)
- ✓ Determine whether a linear equation has one root, no roots, or infinitely many roots
- ✓ Make equations for word problems where quantities are related multiplicatively and the total quantity is known
- ✓ Make equations for before-and-after word problems
- ✓ Use the solution of an equation to find all values that a word problem asks for (not just the value represented by the variable)

There are no Extension Learning Objectives for this unit.

UNIT 10: Final Review (Lessons 105-114)

SUMMARY

Unit 10 reviews the material from all previous units to prepare students for the final exam.

ASSESSMENTS

Quiz Lessons: 110

Final Exam Lesson: 114

ESSENTIAL LEARNING OBJECTIVES

- ✓ Combine like terms in equations
- ✓ Turn a given equation into a linear equation by
 - Ⓐ Moving summands
 - Ⓑ Removing parentheses
 - Ⓒ Using the distributive property
- ✓ Find the root of a linear equation (in the form $ax = b$) by dividing by the coefficient ($x = \frac{b}{a}$)
- ✓ Distribute a negative factor over a sum
- ✓ Evaluate numerical expressions with three operations
- ✓ Simplify an expression containing a variable, then evaluate the simplified expression for a given value of the variable
- ✓ Plot points (with nonzero integer coordinates) on the coordinate plane

EXTENSION LEARNING OBJECTIVES

- ✓ Make equations for before-and-after word problems