The General Approach

1. **Conceptual understanding and discovery learning**
   New ideas are developed first by looking at examples and working special cases, with the student participating actively in the process. (The material is highly interactive.) After working through several examples, the student develops an intuitive sense of the general idea, rule, or method.

2. **Mathematical generalization**
   Beginning with the Basic II curriculum, student learning is carried to the next level of abstraction: once the student has looked through concrete examples and developed a preliminary intuition for the general underlying idea, a rigorous statement of the idea is given and explained. Thus, the RM curriculum combines intuitive understanding of ideas on the one hand with their rigorous formulation on the other, and shows how the latter comes naturally from the former through the process of mathematical generalization.

3. **Rigor**
   All of the rules, definitions, properties, and facts in the curriculum are presented as rigorously as possible. Math vocabulary is used consistently.

4. **Virtual manipulatives**
   Students move around cubes (as well as ten-bars, hundred-plates, and thousand-cubes) when learning whole number place value, color in sections of shapes when studying fractions, and so on. This helps develop conceptual understanding without the logistical difficulties of physical manipulatives.

5. **Computational fluency**
   The curriculum places a strong emphasis on computational fluency. This fluency is achieved not by rote memorization of algorithms, but rather as a natural extension of students’ conceptual understanding of mathematical operations. The conceptual understanding and procedural fluency are developed concurrently, with each supporting the other.

6. **Depth**
   The curriculum focuses strongly on the core areas of arithmetic, geometry, and algebra, focusing on depth over breadth.

7. **Spiraling and scaffolding**
   The curriculum spirals, consistently using new topics to review previous topics; the sequential structure of mathematics – with each topic relying on what came before it – is brought to the foreground. On a related note, the curriculum pervasively uses scaffolding: each topic is presented with an eye to how it will be used in the future.

8. **Higher-order thinking skills**
   Almost every topic has a bank of more difficult problems. Students who master the topic at the “basic” level receive problems at the “advanced” level; if successful with these, students work on problems at the “highest” level. The “advanced” and “highest” level problems are selected to develop higher-order reasoning skills: they are difficult not merely computationally, but also conceptually.

9. **Creative thinking**
   Students solve math riddles, which are problems that require creativity and non-standard thinking. The goal of the RM curriculum is to afford students the opportunity to practice these skills.
Thinking in terms of place value and the composition of a number is the key to understanding the meaning of whole number rounding, comparing whole numbers, and the four basic operations of arithmetic. Children understand the operations – and the associated standard algorithms – in terms of the structure of natural numbers. This knowledge forms a strong foundation for understanding common fractions and decimals.

Mental math is used as a tool for strengthening students’ conceptual understanding of the composition of numbers.

From the very earliest grades, the curriculum introduces algebraic concepts (such as algebraic expressions and solving equations). This prepares students for algebra down the road, and also supports the acquisition of certain arithmetical skills and concepts, such as the properties of multiplication and formulas for distance and price.

For example, perimeter is used as an additional model for the operation of addition – and an important one, since it works for fractions just as well as for natural numbers. Area is used to further develop the understanding of the operation of multiplication, as well as division. Thus, geometry becomes a natural, contributing part of the curriculum, rather than a disconnected topic.

In particular, the number ray is used to illustrate addition and subtraction, as well as the properties of these operations (such as the commutative property of addition and the property of subtracting a number from a sum). When the negative numbers are introduced in 6th grade, the number ray expands to the number line, creating a parallel between the arithmetical expansion of the number system and the geometric expansion of a ray into a line.

Beginning with the Basic II curriculum, subtraction and division are introduced as inverse operations of addition and multiplication, respectively. This more mathematically mature definition significantly widens the class of problems that can be approached with these operations. For example, finding an unknown length of a rectangle given the area and the width now becomes a natural model for division. Such models are particularly important in the Basic II and Basic III curricula, since the conventional models for the operations (e.g., “5 groups of 4 objects” for 5×4) no longer make sense for fractions.

Numerical expressions are extensively used from the very beginning of the Basic I curriculum, and act as a stepping stone to work with algebraic expressions, which in turn lead to equations.

Students learn to use the properties of the operations (e.g., the distributive property of multiplication over addition) to transform numerical and algebraic expressions. This allows them to understand and actively apply these important properties (which in traditional U.S. curricula are taught by rote memorization or entirely omitted), and prepares students for algebra, where without solid skills in algebraic transformations, they will not be able to set up and solve equations.

Students use the properties of the operations for convenient calculation. This means that students are given a numerical expression to evaluate which would be very difficult to evaluate using the order of operations, but quite simple to evaluate if the student first applies certain properties of the operations. This not only serves as a way to make students use the properties, but also develops number sense and mental math skills.
Decimals are taught as a special case (and notation) of common fractions, rather than a separate entity altogether.

Word problems are used extensively, but not just for their own sake. Rather, word problems are a powerful tool for helping students understand the meanings of the mathematical operations. By seeing an operation – say, subtraction – illuminated from different angles and in different contexts, students evolve an abstracted understanding of this operation. The problem sets given in the RM curriculum are carefully selected to achieve this effect.

History

The Reasoning Mind curriculum is essentially based on the Russian curriculum developed in the mid-20th century. This curriculum then went into the foundations of the Chinese curriculum, which in turn was taken as the foundation of the curriculum used in Singapore. Remarkably, most countries that perform well on international math assessments (including Hungary, Bulgaria, Romania, Poland, Belgium, and many others) have a national curriculum that has grown out of the Russian program. Thus, the program has proven itself successful with billions of students.

There is consensus among most Russian educators about the reasons for the success of this approach. Mathematics in this approach is taught in a connected, spiraling fashion; this means that every topic is covered in the context of preceding topics, and especially if they are conceptual prerequisites to the topic.

Problem solving is the key learning activity: theoretical material is given importance, but true conceptual understanding and intuitive command of mathematical ideas comes from solving a series of carefully selected problems. Another reason for the method’s success is that over time, a huge amount of knowledge has been amassed by practitioners of the method as to how certain topics should be best presented to students. Finally, the method places logical, creative, and non-standard thinking as its goal. Students study mathematics to learn how to think, and educators constantly keep this goal in mind.

The Russian curriculum develops both concepts and skills, and emphasizes both discovery learning and mathematical rigor. For this reason, it is widely respected by U.S. math curriculum specialists from across the spectrum of philosophies. RM’s supporters include prominent leaders of both the “constructivist” and the “back to basics” schools of thought – as well as educators in between.

Reasoning Mind took this Russian curriculum as the basis of its approach and modified it to fit the cultural needs of American students. Moreover, RM used the exhibitability of the Internet to enrich the curriculum with animations and an extremely high level of interactivity. The resulting program is very different in the mode of presentation from the Russian curriculum, but at its core one still finds the solid, time-tested instructional practices of the Russian program.